Name: \_\_\_\_\_\_\_\_\_\_\_Blake Williams\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**INEG 3313 - Probability and Statistics**

**Week 5 Homework (100 points)**

For each problem, you can solve by hand or use Excel to help. You need to show all work in either case.

**Question 1 (p184, 4-29).** Suppose that we are testing against . Calculate the *P*-value for the following observed values of the test statistic: (a) , (b) , (c)

1. Find P(Z>2.45)
   * 1. P(Z>2.45)
     2. P(Z>2.45) = 1 – P(Z<=2.45)
     3. P(Z<=2.45) = 0.9929
     4. 1 – 0.9929 = 0.0071
     5. The P value of Z0 = 2.45 is 0.0071
     6. Excel Function: “=1 - NORM.S.DIST(2.45, TRUE)”
2. Find P(Z>1.95)
   * 1. Using same method as a)
     2. 1 – 0.9744 = 0.0256
     3. The P value of Z0 is 0.0256
     4. Excel Function: “=1 - NORM.S.DIST(1.95, TRUE)”
3. Find P(Z>0.25)
   * 1. Using same method as a)
     2. 1 – 0.4013 = 0.5987
     3. The P value of Z0 is 0.5987
     4. Excel Function: “=1 - NORM.S.DIST(-0.25, TRUE)”

**Question 2 (p186, 4-39**). The yield of a chemical process is being studied. From previous experience with this process the standard deviation of yield is known to be 3. The past 5 days of plant operation have resulted in the following yield (in percent %): 91.6, 88.75, 90.8, 89.95, and 91.3. Use .

(91.6+88.75+90.8+89.95+91.3)/5 = 90.48

Sample Mean=90.48

Standard Dev = 3

n = 5

1. Is there evidence that the mean yield is not 90%? Use the *P*-value approach.

μ0​ = 90

Calculating z0 (90.48 – 90) / (3/sqrt(5)) = 0.48/1.3416 = 0.358

Use Two-tailed test

P-value = 2 x (P(Z > abs(z0)

Using Excel function =2 \* (1 - NORM.S.DIST(0.358, TRUE)) which yields 0.7203

Since the P-Value is greater than 0.05 we fail in rejecting the null hypothesis. Meaning there is no evidence to suggest that the mean yield is different from 90%

1. What sample size would be required to detect a true mean yield of 85% with probability of 0.95?

Beta = 1 - .95 = 0.05

Za/2= Z0.05/2=Z0.025=1.96

Z0.05=1.645

Delta = 90 – 85 = 5

N = (( 1.645 + 1.96)/ (5/3))2

N = (2.163)^2

N = 5 // Rounded up from 4.68

Sample size of 5 is required

1. What is the type II error probability if the true mean yield is 92%?

Margin of Error = Z α/2 ​ \* σ/sqrt(n)

Margin of Error = 1.96 \* 3/sqrt(1.34) = 2.63

Critical Value = 90 + 2.63 = 92.63

Z = (92.63 – 92)/1.34 = 0.47

=NORM.S.DIST(0.47, TRUE)

Probability of a Type II error is 68%

1. Find a 95% two-sided CI on the true mean yield.

Mean = 90.48

Margin of Error = 1.96 \* 3/sqrt(1.34) = 2.63

Mean +- Margin of error

90.48 + 2.63 = 93.11%

90.48 – 2.63 = 87.85%

1. Use the CI found in part (d) to test the hypothesis.

Since the value specified by the null hypothesis falls within the confidence interval, we fail to reject the null hypothesis. Leading to the assumption that there is not enough evidence in order to conclude that the true mean yield is different from 90% at the 5% significance rate.

**Question 3 (p197, 4-53**). An article in *Computers in Electrical Engineering* (“Parallel Simulation of Cellular Neural Networks”, 1996, Vol. 22, pp.61-84) considered the speed-up of cellular neural networks for a parallel general-purpose computing architecture. The data follow.

3.775302 3.350679 4.217981 4.030324 4.639692 4.139665

4.395575 4.824257 4.268119 4.584193 4.930027 4.315973

4.600101

1. Is there sufficient evidence to reject the claim that the mean speed-up exceeds 4.0? Assume .

Null hypothesis = u = 4.0

Alternative Hypothesis = u > 4.0

Sample Mean = 4.31322

Sample Standard Deviation = 0.432852

(4.31322 – 4.0) / (0.432852 / sqrt(13) = 2.607

T0.05,12= 1.782

2.607 > 1.782

We reject the Null hypothesis and replace it with the Alternative Hypothesis. So yes there is sufficient evidence.

1. Find a 95% two-sided CI on the mean speed-up time.

E = 2.179 \* (0.432852/sqrt(13) = 0.26157

Upper Limit = Sample Mean + E = 4.31322 + 0.26157 = 4.57479

Lower Limit = Sample Mean – E = 4.31322 - 0.26157 = 4.05165

4.05165 <= u <= 4.57479